1 In a bag, there are only

3 blue beads
4 white beads
and x orange beads.

Jean is going to take at random two beads from the bag.

The probability that Jean will take two beads of the same colour is $\frac{3}{8}$

Find the total number of beads in the bag.
Show clear algebraic working.

Probability of taking two beads of same colour : (let n = total number of beads)

Blue : $\frac{3}{n} \times \frac{2}{n-1} = \frac{6}{n(n-1)}$

White : $\frac{4}{n} \times \frac{3}{n-1} = \frac{12}{n(n-1)}$

Orange : $\frac{n-7}{n} \times \frac{n-8}{n-1} = \frac{(n-7)(n-8)}{n(n-1)}$

Combine : $\frac{6}{n(n-1)} + \frac{12}{n(n-1)} + \frac{(n-7)(n-8)}{n(n-1)} = \frac{3}{8}$ (1)

$6 + 12 + n^2 - 15n + 56 = \frac{3}{8}(n^2 - n)$

$48 + 96 + 8n^2 - 120n + 448 = 3n^2 - 3n$ (1)

$5n^2 - 117n + 592 = 0$ (1)

$n = \frac{117 \pm \sqrt{(-117)^2 - 4(5)(592)}}{2(5)}$

$= \frac{117 \pm 43}{10}$

$= 16, 17.4$

$n = 16$ (1) since no. of beads should be whole number

16

(Total for Question 1 is 4 marks)

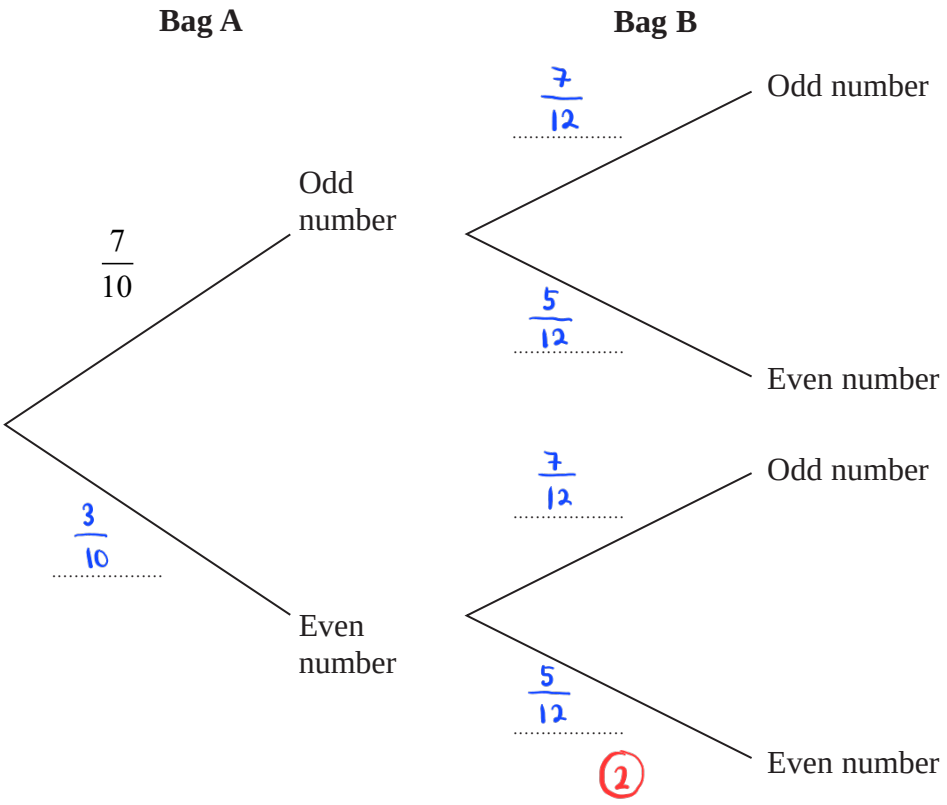
2 Cody has two bags of counters, bag **A** and bag **B**.

Each of the counters has either an odd number or an even number written on it.

There are 10 counters in bag **A** and 7 of these counters have an **odd** number written on them.
There are 12 counters in bag **B** and 7 of these counters have an **odd** number written on them.

Cody is going to take at random a counter from bag **A** and a counter from bag **B**.

(a) Complete the probability tree diagram.



(2)

(b) Calculate the probability that the total of the numbers on the two counters will be an odd number.

To get a total of odd numbers,

① odd + even

② even + odd

① $\frac{7}{10} \times \frac{5}{12} = \frac{7}{24}$ (1)

② $\frac{3}{10} \times \frac{7}{12} = \frac{7}{40}$

Total $= \frac{7}{24} + \frac{7}{40}$ (1)

$= \frac{56}{120}$ (1)

$\frac{56}{120}$

(3)

Harriet also has a bag of counters.
Each of her counters also has either an odd number or an even number written on it.

Harriet is going to take at random a counter from her bag of counters.

The probability that the number on each of Cody's two counters **and** the number on Harriet's counter will all be even is $\frac{3}{100}$

(c) Find the least number of counters that Harriet has in her bag.
Show your working clearly.

Let Harriet's even counter = E

Let Harriet's odd counter = D

P(all even) $= \frac{3}{10} \times \frac{5}{12} \times \frac{E}{E+D} = \frac{3}{100}$ (1)

$\frac{E}{E+D} = \frac{0.03}{0.125}$

$\frac{E}{E+D} = 0.24$ (1)

$E = 0.24E + 0.24D$

$0.76E = 0.24D$

$E = \frac{6}{19}D$ → 6 E and 19 D

∴ Least number of counter is 6 + 19 = 25 (1)

25

(3)

(Total for Question 2 is 8 marks)

3 Pippa has a box containing N pens.

There are only black pens and red pens in the box.
The number of black pens in the box is 3 more than the number of red pens.

Pippa is going to take at random 2 pens from the box.

The probability that she will take a black pen **followed** by a red pen is $\frac{9}{35}$

Find the possible values of N.
Show clear algebraic working.

Let black pens = B
red pens = R

$B = R + 3$

$B + R = N$

$(R+3) + R = N$

$2R + 3 = N$ (1)

$P(B) \times P(R) = \frac{9}{35}$

$\frac{R+3}{N} \times \frac{R}{N-1} = \frac{9}{35}$

$\frac{R+3}{2R+3} \times \frac{R}{2R+2} = \frac{9}{35}$

$\frac{R^2 + 3R}{4R^2 + 10R + 6} = \frac{9}{35}$ (1)

$35R^2 + 105R = 36R^2 + 90R + 54$ (1)

$R^2 - 15R + 54 = 0$ (1)

$R = \frac{15 \pm \sqrt{(-15)^2 - 4(1)(54)}}{2}$

$= \frac{15 \pm \sqrt{9}}{2}$

$= \frac{15 \pm 3}{2}$

$R = 9$ or 6

$N = 2(9) + 3$ or $2(6) + 3$

$= 21$ or 15 (1)

21 , 15

(Total for Question 3 is 5 marks)

4 A bag contains n beads.
6 of the beads are red and the rest are blue.

Ravi is going to take at random 2 beads from the bag.

The probability that the 2 beads will be of the same colour is $\frac{9}{17}$

Using algebra, and showing each stage of your working, calculate the value of n.

blue = $n-6$

red = 6

Probability same colour: $P(RR) + P(BB)$

Probability both is red = $P(RR) = \frac{6}{n} \times \frac{5}{n-1}$ (1)

$= \frac{30}{n(n-1)}$

Probability both is blue = $P(BB) = \frac{n-6}{n} \times \frac{(n-6-1)}{(n-1)}$

$= \frac{(n-6)(n-7)}{n(n-1)}$ (1)

$= \frac{n^2-13n+42}{n(n-1)}$

Probability same colour: $P(RR) + P(BB)$

$\frac{9}{17} = \frac{30}{n(n-1)} + \frac{n^2-13n+42}{n(n-1)}$ (1)

$\frac{9}{17} = \frac{n^2-13n+72}{n^2-n}$

$9(n^2-n) = 17(n^2-13n+72)$

$9n^2-9n = 17n^2-221n+1224$

$0 = 8n^2-212n+1224$

$0 = 2n^2 - 53n + 306$ (1)

$(2n-17)(n-18) = 0$ (1)

$n = \frac{17}{2}$ or 18

Since n must be an integer, our n must be 18.

$n =$ 18 (1)

(Total for Question 4 is 6 marks)

5 Elliot has x counters.

Each counter has one red face and one green face.

Elliot spreads all the counters out on a table and sees that the number of counters showing a red face is 5

Elliot then picks at random one of the counters and turns the counter over.
He then picks at random a second counter and turns the counter over.

The probability that there are still 5 counters showing a red face is $\frac{19}{32}$

Work out the value of x
Show clear algebraic working.

To get 5 counters still showing red face :

① First pick (R) + second pick (G)

② First pick (G) + second pick (R)

① $\frac{5}{x} \times \frac{(x-4)}{x} = \frac{5x-20}{x^2}$ (1)

② $\frac{(x-5)}{x} \times \frac{6}{x} = \frac{6x-30}{x^2}$

① + ② $= \frac{19}{32}$

$\frac{5x-20+6x-30}{x^2} = \frac{19}{32}$ (1)

$11x - 50 = \frac{19}{32}(x^2)$

$32(11x-50) = 19x^2$

$19x^2 - 352x + 1600 = 0$ (1)

$(19x - 200)(x-8) = 0$ (1)

$x = \frac{200}{19}$ or $x = 8$

$x =$ 8 (1)

so $x = 8$ since $\frac{200}{19}$ is not a whole number

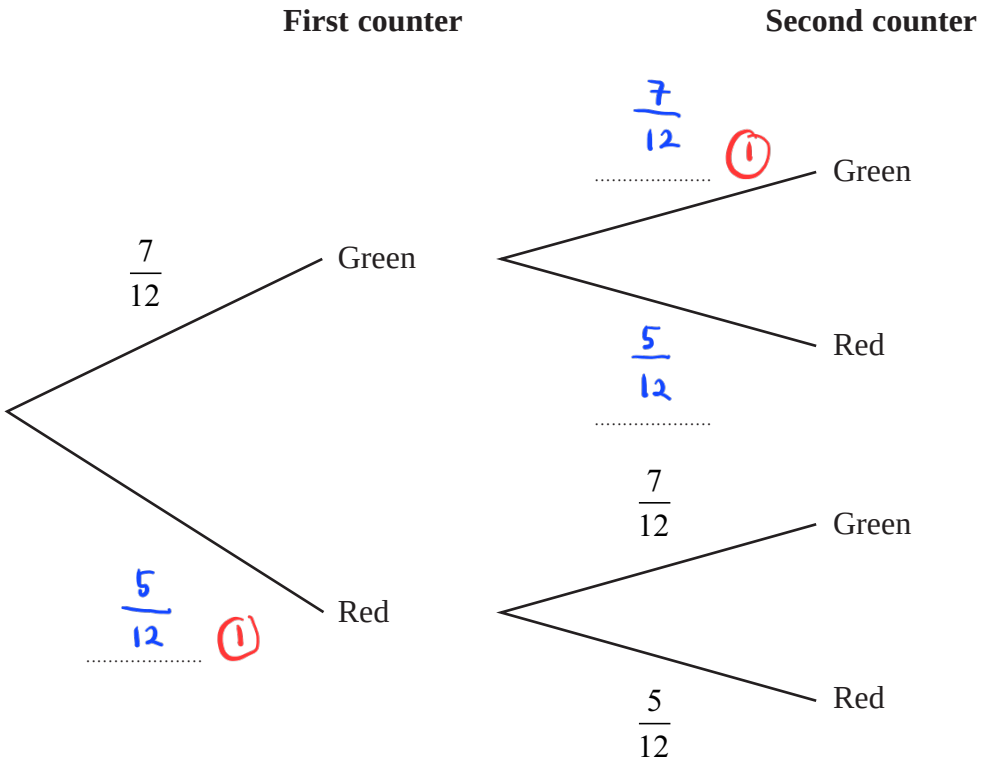
(Total for Question 5 is 5 marks)

6 Hector has a bag that contains 12 counters.
There are 7 green counters and 5 red counters in the bag.

Hector takes at random a counter from the bag.
He looks at the counter and puts the counter back into the bag.

Hector then takes at random a second counter from the bag.
He looks at the counter and puts the counter back into the bag.

(a) Complete the probability tree diagram.



(2)

(b) Work out the probability that both counters are red.

$\frac{5}{12} \times \frac{5}{12} = \frac{25}{144}$

$\frac{25}{144}$

(2)

Meghan has a jar containing 15 counters.
There are only blue counters, green counters and red counters in the jar.

Hector is going to take at random one of the counters from his bag of 12 counters.
He will look at the counter and put the counter back into the bag.

Hector is then going to take at random a second counter from his bag.
He will look at the counter and put the counter back into the bag.

Meghan is then going to take at random one of the counters from her jar of counters.
She will look at the counter and put the counter back into the jar.

The probability that the 3 counters each have a different colour is $\frac{7}{24}$

(c) Work out how many blue counters there are in the jar.

① RG and GR : $\frac{7}{12} \times \frac{5}{12} \times 2$ ①

P(B) from jar

① and blue : $2 \times \frac{7}{12} \times \frac{5}{12} \times y = \frac{7}{24}$

$y = \dfrac{\frac{7}{24}}{2 \times \frac{7}{12} \times \frac{5}{12}} = \frac{3}{5}$ ①

$\frac{3}{5} \times 15$ counters = 9 blue counters ①

9

.....

(3)

(Total for Question 6 is 7 marks)

7 In a bag, there are only red counters, blue counters, green counters and yellow counters.

The total number of counters in the bag is 80

In the bag

the number of red counters is $x + 7$
the number of blue counters is $x - 11$
the number of green counters is $3x$

Jude takes at random a counter from the bag.

The probability that he takes a red counter is $\frac{1}{4}$

Work out the probability that Jude takes a yellow counter.

red $= \frac{x+7}{80} = \frac{1}{4}$ (1)

$x = 20 - 7$

$= 13$ (1)

yellow $= 80 - (13+7) - (13-11) - (3\times13)$ (1)

$= 80 - 20 - 2 - 39$

$= 19$

$\frac{19}{80}$ (1)

(Total for Question 7 is 4 marks)

8 Moeen has a biased 6-sided dice.

The table gives information about the probability that, when the dice is thrown, it will land on each number.

Number	1	2	3	4	5	6
Probability	x	0.15	0.5	y	0.13	0.03

(a) Show that $x + y = 0.19$

$x + y + 0.15 + 0.5 + 0.13 + 0.03 = 1$ (1)

$x + y + 0.81 = 1$

$x + y = 1 - 0.81 = 0.19$ (1)

(2)

Given that $3x - y = 0.09$

and $x + y = 0.19$

(b) work out the value of x and the value of y
Show clear algebraic working.

$x = 0.19 - y$

$3(0.19 - y) - y = 0.09$ (1)

$0.57 - 3y - y = 0.09$

$-4y = -0.48$

$y = 0.12$

$x = 0.19 - 0.12$ (1)

$= 0.07$

(1)

$x =$ 0.07

$y =$ 0.12

(3)

(Total for Question 8 is 5 marks)